Robust Procurement Contracts

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Abstract: We study a procurement problem where both a principal and an agent share uncertainty regarding the cost at the outset. After being offered a contract, the agent privately observes an informative signal of the marginal cost. The principal neither knows the signal distribution nor has a prior belief about possible signal distributions. We characterize a procurement contract that is robust to the principal's uncertainty about the agent's information structure. The principal's worst distribution either fully reveals to the agent that the cost is low or makes him just pessimistic enough to reject the contract. In the former case, the agent accepts the contract and produces less than what he would have without the signal.

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1. Introduction

In contractual relationships, it is typical for one party to acquire payoff-relevant information *after* a contract is offered but *before* a decision of whether to accept/reject it is made. For instance, consider a standard procurement with unknown production costs, where a principal chooses an output that the agent has to deliver and a corresponding transfer. After the agent is offered a contract but before he accepts it, he can run forecasts or obtain other informative signals correlated with the true production cost. This information allows the agent to better estimate his payoff from the contract and, as a result, obtain some rent which typically leads to inefficiency in production. A large literature analyzes optimal contracts in this environment assuming the principal knows either exactly which signal the agent can acquire or what distribution a signal is drawn from.[1]

What if the principal neither knows the type of information the agent has access to nor has a prior belief about possible distributions this information comes from? For example, consider the construction industry. Before a supplier accepts a contract, the employed architects use advanced software to estimate all the costs and the materials required to complete the project, and the buyer might not be aware of the exact techniques being used (see, e.g., Bajari and Tadelis (2001)).[2] What is the principal's worst information structure that minimizes her expected profit? What profit can the principal guarantee herself in this environment?

In an otherwise standard procurement model, we derive an optimal contract that is robust to the principal's uncertainty regarding the agent's information structure. We characterize the worst distribution for the principal and a novel trade-off that emerges in this environment. While a higher output and a smaller transfer deliver a higher surplus if the agent accepts the contract, they also make it more likely the agent rejects the contract. Therefore, the principal trades off the surplus she receives if the agent agrees to deliver the output with the chance that the agent accepts the contract. Our key contribution is to study the effect of the uncertainty regarding the agent's information structure on the optimal contract and, in particular, on distortions in output.

This paper builds on two strands of the literature. First, it is related to the literature on the application of the principal-agent model to information gathering.[3] Most of the papers assume that the agent obtains a signal from a distribution that is known to the principal (see Terstiege (2016a), Khalil et al. (2020), and references therein).[4] We study an extreme scenario where the principal neither knows the signal distribution nor has any prior beliefs regarding possible signal distributions. We show how the concavification approach can be used to characterize the principal's worst distribution in this environment.

[1] See Crémer and Khalil (1992) for an early paper and Terstiege (2012) for more recent references.
[2] Another example is healthcare procurement. A third-party payer might not be aware of the information sources available to a physician who decides whether or not to admit a patient. As Dranove (1987) explains, based on characteristics observable upon admission, physicians can know whether a patient will be easy or difficult to diagnose. Physicians have private information when diagnosing a patient because they privately observe some patient characteristics the third-party payer might not be aware of.
[3] Early papers are Crémer and Khalil (1992, 1994) and Crémer et. al. (1998).
[4] In Lewis and Sappington (1994), the principal chooses the distribution the agent obtains a signal from.

Also, our paper contributes to the literature that studies information structure in buyer-seller relationships. Hinnosaar and Kawai (2020) study an optimal buyer-seller contract in an environment when refunds are feasible. We contribute to this literature by making the output endogenous and by characterizing the worst distribution for the buyer.

2. Model

Consider a standard procurement problem. There is a principal (she) and an agent (he). The principal wants the agent to deliver some quantity $q \geq 0$ of output. The value of production to the principal is measured by a differentiable, strictly increasing, and strictly concave utility function $V()$.

The disutility of producing q units is cq, and the agent bears this disutility. If q units of output are produced and the principal pays the agent t dollars, the payoff to the principal is $V(q) - t$, and the net benefit of the agent is $t - cq$. We refer to a combination of output and a transfer, (q, t), as a contract.

Initially, the principal and the agent do not know the precise value of c but share a common prior about it, which is supported on $[c_L, c_H]$ with some commonly known *cdf*, where $c_H > c_L > 0$. The prior expected cost is denoted by c_0:

$$c_0 = \mathbb{E}_0 c. \tag{1}$$

After the agent is offered a contract by the principal but before he accepts/rejects it, he receives a private signal of the marginal cost.[5] The principal neither knows the agent's signal distribution nor has a prior belief about possible signal distributions. She only knows that the signal satisfies the Bayes' rule: the expected posterior beliefs must equal to the prior.

The agent's signal leads to posterior beliefs with the expected cost β that is a random variable drawn from a distribution function F, where:

$$F \in \mathcal{F} \equiv \{F: \mathbb{E}_F[\beta] = c_0\}. \tag{2}$$

A distribution F over posterior expected costs is therefore used to represent the agent's relevant information structure after he has obtained the signal. We denote by $\mu(\beta)$ the probability that the agent's posterior expected cost is β.

The agent's outside option is assumed to be zero. Therefore, given any posterior beliefs β, the agent accepts the contract if $t - \beta q > 0$, and rejects it otherwise.

3. Analysis

For any agent's signal distribution $F \in \mathcal{F}$, we denote the principal's expected profit when she offers a contract (q, t) to the agent by $\pi\big((q, t)|F\big)$. A contract guarantees at least some profit level $\underline{\pi}$ to the principal if her expected profit is at least $\underline{\pi}$ for any possible distribution $F \in \mathcal{F}$.

[5] Since the only decision the agent makes after learning the signal is to accept/reject the contract, our results remain intact if the contract is offered after the realization of the signal.

Definition 1. The contract (q,t) *guarantees at least* profit $\underline{\pi}$ if

$$\pi\big((q,t)|F\big) \geq \underline{\pi} \text{ for } F \in \mathcal{F} \equiv \{F: \mathbb{E}_F[\beta] = c_0\}.$$

The principal chooses a contract (q,t) to maximize $\underline{\pi}(q,t)$, defined as

$$\underline{\pi}(q,t) \equiv \min_{F\in\mathcal{F}} \pi\big((q,t)|F\big). \tag{3}$$

We denote by π^* the principal's best-guaranteed profit defined next.

Definition 2. π^* is the best-guaranteed profit if $\pi^* \equiv \max_{(q,t)} \underline{\pi}(q,t)$.

Let $\gamma\big(\beta|(q,t)\big)$ denote the principal's expected profit when she offers a contract (q,t) and the agent's posterior expected cost is β. Consequently,

$$\pi\big((q,t)|F\big) = \mathbb{E}_F\big[\gamma\big(\beta|(q,t)\big)\big]. \tag{4}$$

Therefore, the principal's expected profit given the agent's posterior β becomes:

$$\gamma\big(\beta|(q,t)\big) = \begin{cases} V(q) - t, & \beta < \frac{t}{q} \\ 0, & \beta \geq \frac{t}{q} \end{cases}. \tag{5}$$

To derive the exact value of $\underline{\pi}(q,t)$ we apply the techniques from Bayesian persuasion problems.[6] Using the terminology of Kamenica and Gentzkow (2011), the agent is a "Receiver" and Nature is a "Sender". The Nature, for a given contract (q,t), chooses the distribution F to minimize $\gamma\big(\beta|(q,t)\big)$ with the only condition that the signal has to satisfy the Bayes' rule, i.e., $F \in \mathcal{F}$. To apply the conventional Bayesian persuasion approach directly, we will say that Nature chooses the distribution F to maximize the negative of the principal's expected profit, $-\gamma\big(\beta|(q,t)\big)$, subject to $F \in \mathcal{F}$. After obtaining the signal from Nature, the agent takes one of the two possible actions – accepts or rejects the contract.

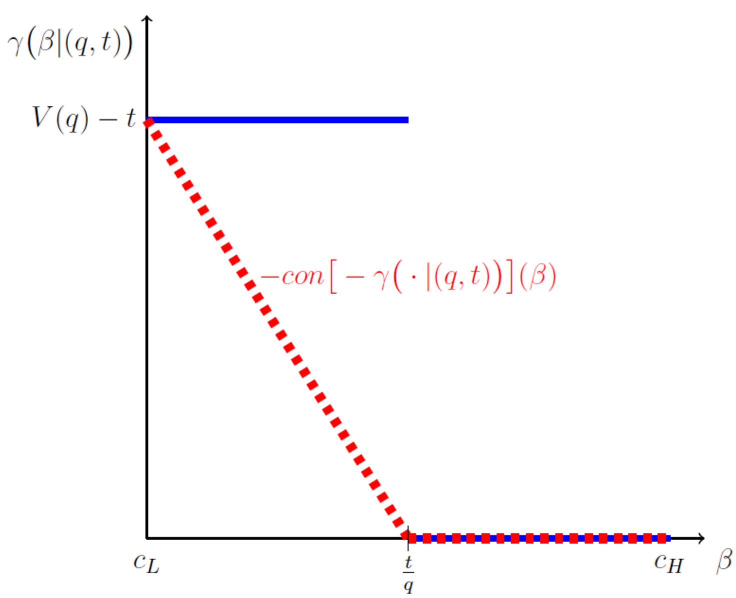

Figure 1. The principal's profit from a contract (q,t).

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[6] See Aumann and Maschler (1995) and Kamenica and Gentzkow (2011).

Given the prior expected cost c_0, the value of $\underline{\pi}(q,t)$ then can be derived using the concavification approach:[7]

$$\underline{\pi}(q,t) = -con\left[-\gamma(\cdot\,|(q,t))\right](c_0),$$

where $-con\left[-\gamma(\cdot\,|(q,t))\right](\beta)$ is represented by the red dashed line in the Figure 1.

3.1. The Worst Distribution for the Principal

We now characterize the worst distribution for the principal. There are two cases to consider depending on whether the agent rejects or accepts the contract given his prior beliefs. First, when the prior expected cost is high so that the agent rejects the contract if he did not receive any additional signal, i.e., $c_0 \geq \frac{t}{q}$. In this case, the worst distribution for the principal is when the agent's signal does not disclose any additional information, i.e., $\beta = c_0$ with probability $\mu(\beta = c_0) = 1$.

Second, when the prior expected cost is low so that the agent would accept the contract if he did not receive any additional signal, i.e., $c_0 < \frac{t}{q}$. In this case, the Nature maximizes $\Pr\left(\beta \geq \frac{t}{q}\right) = 1 - \Pr\left(\beta < \frac{t}{q}\right) = 1 - F\left(\frac{t}{q}\right)$ subject to the constraint that the expected posterior must equal to the prior. Formally, the Nature's problem is:

$$\max_{F\in\mathcal{F}} 1 - F\left(\frac{t}{q}\right).$$

The (unique) worst distribution for the principal is characterized as

$$\mu(\beta = c_L) = \frac{\frac{t}{q}-c_0}{\frac{t}{q}-c_L} \text{ and } \mu\left(\beta = \frac{t}{q}\right) = 1 - \frac{\frac{t}{q}-c_0}{\frac{t}{q}-c_L} = \frac{c_0-c_L}{\frac{t}{q}-c_L}.$$

Intuitively, the worst signal distribution for the principal makes it so that the agent is fully convinced the cost is low when he accepts the contract and is (almost) indifferent between accepting and rejecting when he rejects.

3.2 The Optimal Contract

To derive the optimal contract, we first present the equation for the red dashed line in Figure 1, which is given by:

$$-con\left[-\gamma(\cdot\,|(q,t))\right](\beta) = \begin{cases} (V(q)-t)\left(1-\frac{q}{t}\beta\right), & \beta \leq \frac{t}{q} \\ 0, & \beta > \frac{t}{q} \end{cases}. \tag{6}$$

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[7] The concave closure of any function f is defined as

$$con[f](x) = \sup\{y|(x,y) \in co(f)\},$$

where $co(f)$ is the convex hull of the graph of f.

The concavification approach then dictates the principal is choosing a contract (q, t) to maximize (6) evaluated at $\beta = c_0$, $-con\left[-\gamma\left(\cdot \mid (q,t)\right)\right](c_0)$.

The principal's problem is then equivalent to choosing a contract (q, t) to maximize

$$(V(q) - t)\left(1 - \frac{q}{t}c_0\right)$$

$$\text{s.t. } t - c_0 q \geq 0.$$

When designing the optimal contract, the principal faces the following trade-off: While a higher output q and a smaller transfer t deliver a higher surplus if the agent accepts the contract, they also make it more likely the agent rejects the contract. This follows directly from the principal's objective function. The first term, $(V(q) - t)$, is the surplus the principal gets if the agent accepts the contract. It is strictly increasing in q and decreasing in t. The second term, $\left(1 - \frac{q}{t}c_0\right)$ reflects the probability that the agent accepts the contract. It is strictly decreasing in q and increasing in t.

We describe the main result in Proposition 1 below.

Proposition 1. ***The Optimal Contract*** $(\boldsymbol{q}^*, \boldsymbol{t}^*)$**:**

$$t^* = q^* V'(q^*) \text{ and } V'(q^*) = c_0 + \frac{\left(V(q^*) - q^* V'(q^*)\right)c_0}{V'(q^*)q^*}.$$

We next discuss the distortions in the optimal output. Since $V(q^*) - q^* V'(q^*) > 0$ and $V'(q^*) > 0$, it follows that

$$V'(q^*) > c_0.$$

Denoting by q_0 and by q_L the efficient output when the marginal cost is c_0 and c_L, respectively:

$$V'(q_0) = c_0 \text{ and } V'(q_L) = c_L,$$

we conclude that

$$q^* < q_0 < q_L.$$

Therefore, the principal asks the agent to produce less than what she would have asked without the additional signal, i.e., $q^* < q_0$. The reason for this downward distortion in the output is the trade-off described above. That is, the output offered by the principal cannot be very efficient since then the contract would be rejected with high probability.

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